Application of Homotopy Perturbation Method to A nonlinear Focusing Manakov System

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Abstract

In this paper, the homotopy perturbation method (HPM) is developed to solve the focusing Manakov system of coupled nonlinear Schrödinger equations in one space variable and two space variables. The advantages of this method are, fast convergent, does not require discretizations of space-time variables and does not require to solve the resultant nonlinear system of discrete equations. Numerical examples are shown to demonstrate the accuracy and capability of the method. The accuracy of the method is verified by ensuring that the conserved quantities remain almost constant.

Key words : Manakov system in one and two space variables, Homotopy perturbation method.

AMS Classification : 35Q51, 35Q70 , 65M30

Introduction

In recent years, a growing interest towards the applications of the homotopy technique in nonlinear problems has been devoted by engineering practice [13]. The HPM , which proposed by J. H. He (see [10]-[16] and the references sited therein), have several attractive properties (see [1-3], [27]), it is a fast convergent method, does not require discretizations of space-time variables and does not require to solve the resultant nonlinear system of discrete equations. The main advantage of HPM is that it can be continuously deform a simple problem easy to solve into difficult problem under study. Also, Homotopy perturbation method (HPM) can overcome the difficulties arising in calculation of Adomian polynomials in Adomian decomposition method (see [5], and the references sited therein). On the other hand the focusing Manakov system in one space variable and two space variables are an integrable system of coupled nonlinear Schrödinger equations (see [4]-[6-9], [17-19], [22-26]) which models the propagation of the average fields of a polarized wave in a randomly birefringent optical fiber. The qualitative nature of the unstable manifolds of linearly unstable plan wave solutions has been examined in details in [6,7]. For more details on the coupled nonlinear Schrödinger equations and the focusing Manakov system see [4], [17], and the references therein. For high order methods for solving coupled nonlinear partial differential equations see [20-21], [28] and the references sited therein

In this paper we develop HPM to solve the Manakov system in the one-dimensional of the following form:

$$i u_t + \frac{1}{2} u_{xx} + q(|u|^2 + |v|^2) u = 0 , \qquad x \in R, t \ge 0 \tag{1}$$

$$i v_t + \frac{1}{2} v_{xx} + q(|u|^2 + |v|^2) v = 0, \tag{2}$$

With an initial conditions $u(x,0) = u^0(x)$, $v(x,0) = v^0(x)$ and homogenous boundary conditions. Note that the assumption of homogenous boundary conditions is for simplicity only and is not essential: the method can be easily designed for arbitrary domain and non-homogenous boundary conditions. The two-dimensional version of Manakov system is

$$i u_t + \Delta u + q(|u|^2 + |v|^2) u = 0 , \quad (x,y) \in R^2, t \ge 0 \tag{3}$$

$$i v_t + \Delta v + q(|u|^2 + |v|^2) v = 0 , \tag{4}$$

with an initial conditions $u(x,y,0) = u^0(x,y)$, $v(x,y,0) = v^0(x,y)$ and homogenous boundary conditions, where $\Delta := \partial_{xx} + \partial_{yy}$ is the Laplacian operator in two-dimensional. Both systems (1)-(2) and (3)-(4) were first analyzed in [14] (and hence are often referred to as the Manakov system). It is known that the Manakov system is completely integrable, i.e., they can be solved by the inverse scattering method [22]. There has been a lot of previous work on the solitary wave equations to system (1)-(2) under the infinite boundary condition $u \to 0, v \to 0$ at $|x| \to \infty$ (see [17]-[20] and the references therein). An important goal of the present work is to show that the developed homotopy perturbation method (HPM) is applicable to solve numerically the focusing Manakov system, i.e., $q = +1$, in one space variable and two space variables.

2. Analysis of the Homotopy Perturbation Method

In this section, we present the analysis of the homotopy perturbation method for solving the following non-homogeneous, nonlinear coupled system of partial differential equations

$$L_1 u(x,t) + N_1(u(x,t), v(x,t)) = f(x,t) , \tag{5}$$

$$L_2 v(x,t) + N_2(u(x,t), v(x,t)) = g(x,t) , \tag{6}$$

where L_1, L_2 are linear differential operators with respect to time t and N_1, N_2 are non-linear operators and $f(\boldsymbol{x},t)$, $g(\boldsymbol{x},t)$ are arbitrary (smooth) nonlinear given functions.

According to the homotopy perturbation method, we construct the following simple homotopy

$$H_1(u,v,p) = (1-p) L_1\, u(x,t) + p[L_1\, u(x,t) + N_1(u(x,t), v(x,t)) - f(x,t)] = 0 , \tag{7}$$

$$H_2(u,v,p) = (1-p) L_2\, v(x,t) + p[L_2\, v(x,t) + N_2(u(x,t), v(x,t)) - g(x,t)] = 0 , \tag{8}$$

or

$$H_1(u,v,p) = L_1 u(x,t) + p[N_1(u(x,t), v(x,t)) - f(x,t)] = 0 , \tag{9}$$

$$H_2(u,v,p) = L_2 v(x,t) + p[N_2(u(x,t), v(x,t)) - g(x,t)] = 0 , \tag{10}$$

Where $p \in [0,1]$ is an embedding parameter. In case p=0, equations (9) and (10) become linear equations of the form $L_1\, u(x,t) = 0$ and $L_2\, v(x,t) = 0$ which can be easily solved ; also when p=1, equations (9) and (10) turns out to be the original equations (5) and (6). In the view of homotopy perturbation method, we use the homotopy parameter p to expand the solutions as follows:

$$u(x,t) = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + . . . \qquad (11)$$

$$v(x,t) = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + . . . \qquad (12)$$

The approximate solutions can be obtained by setting $p = 1$ in equations (11) and (12):

$$u(x,t) \cong U(x,t) = u_0 + u_1 + u_2 + u_3 + . . . \qquad (13)$$

$$v(x,t) \cong V(x,t) = v_0 + v_1 + v_2 + v_3 + . . . \qquad (14)$$

Substituting from (11)-(12) into (9)-(10) and equating the terms with the identical powers of p, we can obtain a series of linear equations. These linear equations are easy to solve by using Mathematica software or by setting a computer code to get as many equations as we need in the calculation of the numerical as well as explicit solutions.

3. Application and Numerical Results

In this section the HPM is applied for solving the focusing Manakov system in one space variable and then for two-space variable. Numerical results are given, also the accuracy of the method is verified by ensuring that the conserved quantities.

3.1 Focusing Manakov System in one Space Variable

Consider the system (1)-(2) in the region $R = [x_L < x < x_R] \times [t > 0]$ with its boundary ∂R which consists of the ordinates $x = x_L, x = x_R$ and the axis $t \geq 0$. According to the homotopy perturbation methodology, we construct following simple homotopies:

$$u_t + p[-\frac{1}{2} i\, u_{xx} - qi\ (|u|^2 + |v|^2)\, u\,] = 0, \qquad (15)$$

$$v_t + p[-\frac{1}{2} i\, v_{xx} - qi\ (|u|^2 + |v|^2)\, v\,] = 0 \qquad (16)$$

where $p \in [0,1]$ is an embedding parameter, we use it to expand the solutions in the following form:

$$u(x,t) = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + . . . \qquad (17)$$

$$v(x,t) = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + . . . \qquad (18)$$

The approximate solution can be obtained by setting $p = 1$ in equations (17)-(18):

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + . . . \text{ and } \quad v(x,t) = v_0 + v_1 + v_2 + v_3 + . . . \qquad (19)$$

Now substituting from (17)-(18) into (15)-(16) respectively, and equating the terms with the identical powers of p, we can obtain series of linear equations. These linear equations are easy to solve by using Mathematica software or by setting a computer code to get as many equations as we need in the calculation of the numerical as well as explicit solutions. Here we only write the first few linear equations:

$$p^0: \ \dot{u}_0(x,t) = 0, \quad (20\text{-i})$$

$$\dot{v}_0(x,t) = 0, \quad (20\text{-ii})$$

$$p^1: \ \dot{u}_1(x,t) = \frac{1}{2} i\, u_{0xx} + i\, u_0(|u_0|^2 + |v_0|^2), \quad (21\text{-i})$$

$$\dot{v}_1(x,t) = \frac{1}{2} i\, v_{0xx} + i\, v_0(|u_0|^2 + |v_0|^2), \quad (21\text{-ii})$$

$$p^2: \ \dot{u}_2(x,t) = \frac{1}{2} i\, u_{1xx} + i\, u_0(2|u_0||u_1| + 2|v_0||v_1|) + i\, u_1(|u_0|^2 + |v_0|^2), \quad (22\text{-i})$$

$$\dot{v}_2(x,t) = \frac{1}{2} i\, v_{1xx} + i\, v_0(2|u_0||u_1| + 2|v_0||v_1|) + i\, v_1(|u_0|^2 + |v_0|^2), \quad (22\text{-ii})$$

Solutions of equations (20) can be calculated by using the following initial conditions [22] :

$$u_0(x,t) = u(x,0) = a_0(1 - \varepsilon \cos(\sqrt{2}\alpha\, x)), \qquad v_0(x,t) = v(x,0) = b_0(1 - \varepsilon \cos(\sqrt{2}\alpha\, x)),$$

Where a_0, b_0 are the initial amplitudes of the two perturbed periodic waves, respectively and $\varepsilon << 1$ is a small parameter and it represents the strength of the perturbation and α is the wave number of the perturbation. Then, we can derive solutions of (21) in the following form :

$$u_1(x,t) = \int_0^t [\frac{1}{2} i\, u_{0xx} + i\, u_0(|u_0|^2 + |v_0|^2)]dt$$

$$= i\, a_0 t\, (2\alpha^2 \varepsilon \cos(\sqrt{2}\alpha\, x) - q(a_0^2 + b_0^2)\, (-1 + \varepsilon \cos(\sqrt{2}\alpha\, x))^3),$$

$$v_1(x,t) = \int_0^t [\frac{1}{2} i\, v_{0xx} + i\, v_0(|u_0|^2 + |v_0|^2)]dt$$

$$= i\, b_0 t\, (2\alpha^2 \varepsilon \cos(\sqrt{2}\alpha\, x) - q(a_0^2 + b_0^2)\, (-1 + \varepsilon \cos(\sqrt{2}\alpha\, x))^3).$$

Therefore, the complete approximate solution can be readily obtained by the same iterative process. Now, to illustrate the advantages and the accuracy of the homotopy perturbation method for solving focusing Manakov system in one space variable, we have applied the method by using the first order perturbation only, i.e. the approximate solutions are

$$u(x,t) \cong U(x,t) = u_0(x,t) + u_1(x,t), \quad (23)$$

$$v(x,t) \cong V(x,t) = v_0(x,t) + v_1(x,t) \quad (24)$$

The numerical simulations are represented in Figure (1) for the approximated wave solutions $|u(x,t)|$ which represented in the top, and $|v(x,t)|$, which represented in the bottom of the figure, at different time values from $t = 0$ to $t = 20$ in the region $0 \le x \le 115$. The numerical results are obtained by using first perturbation term of equation (19) where $a_0 = 0.08$, $b_0 = 0.1$, $\alpha = 0.05$, $\varepsilon = 0.05$

We achieved a very good approximation for the solution of the system. It is evident that the overall errors can be made smaller by adding new terms from the iteration formulas.

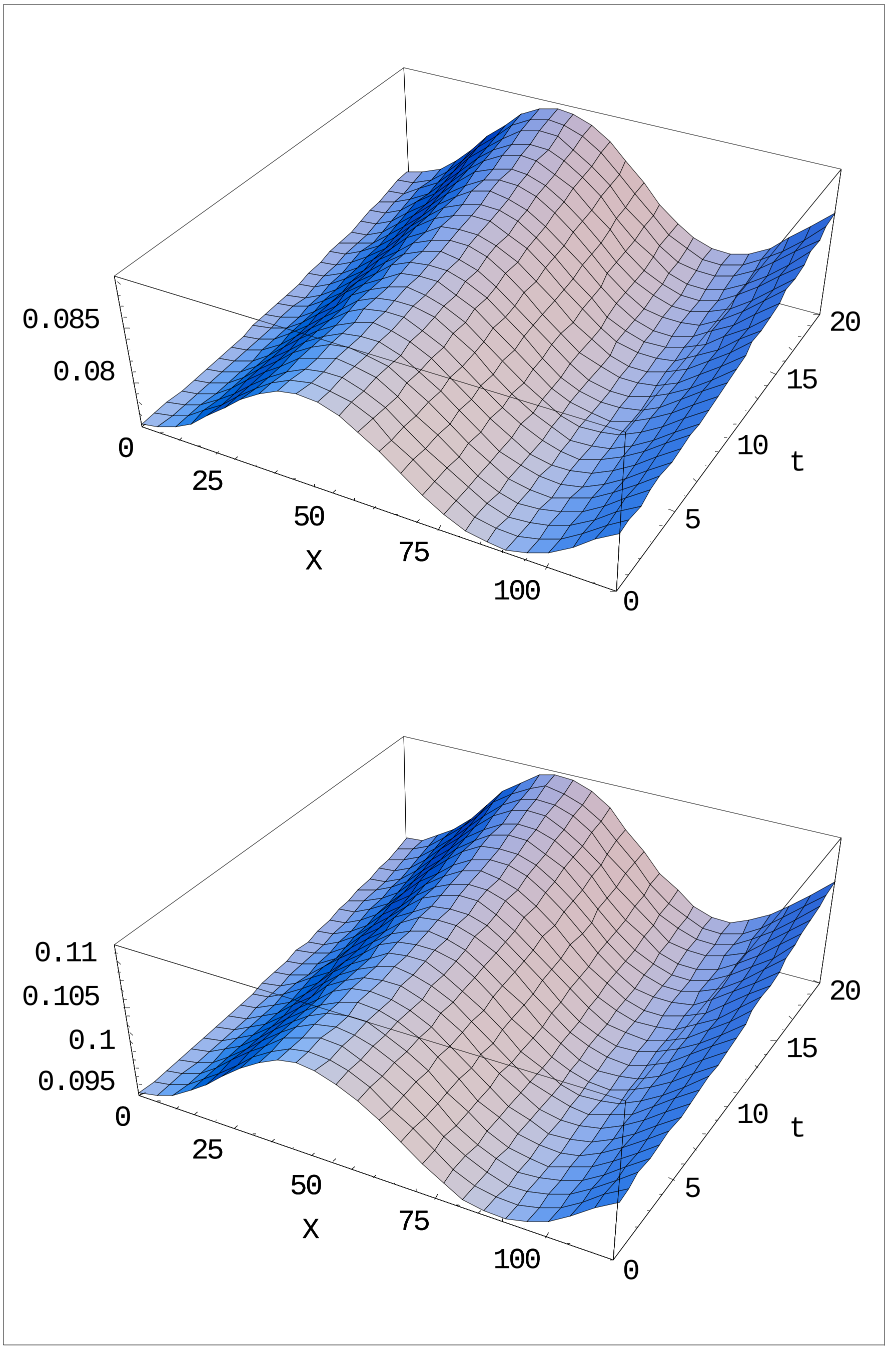

Figure 1: Long-time evolution of the wave solution |u(x, t)| and |v(x, t)|

3.2. Conserved quantities in one space variable

In order to verify whether the proposed methodology lead to higher accuracy, we will use the same procedure as in Sun and Qin [19] which emphasize that a good numerical scheme should have excellent long-time numerical behavior, as well as energy conservation property. To monitor the accuracy of the homotopy perturbation method, we consider the following two conserved quantities,

$$E(u) = \int_{-s/2}^{S/2} |u(x,t)|^2 \, dx$$

and

$$E(v) = \int_{-s/2}^{S/2} |v(x,t)|^2 \, dx$$

Where $s = (2\pi/\alpha) = 40\pi$ (for $\alpha = 0.05$) is the spatial period of the solution [21]. Table 1 shows $E(u)$ and $E(v)$ for various times. The nearly constant values of both $E(u)$ and $E(v)$ show that the method is working well.

Time	E(u)	E(v)
3	0.824864	1.28885
6	0.831176	1.29871
9	0.841697	1.31515
12	0.856426	1.33817
15	0.875363	1.36776
18	0.898509	1.39513

Table 1

Following the stability analysis suggested by Tan and Boyd [23] , the wave solution is linearly stable only if the perturbation wave number α is above the critical value $\alpha_c = \sqrt{2(a_0^2 + b_0^2)}$; otherwise the wave solution is unstable. For the our chose of the constants $a_0 = 0.08$, $b_0 = 0.1$, $\alpha = 0.05$, we find that α_c=0.181108 , therefore, the wave solution in this case is unstable. The amplitude of u and v undergoes oscillations between the near-uniform state and the one-hume state .

3.3 Focusing Manakov system in two space variables

In order to develop a numerical simulation for solving the system (3)-(4) by the HPM in the region $R = [x_L < x < x_R] \times [y_L < y < y_R] \times [t > 0]$ with its boundary ∂R which consists of the ordinates $x = x_L, x = x_R, y = y_L, x = y_R$ and the axis $t \geq 0$. According to the homotopy perturbation method, we construct following simple homotopies:

$$u_t + p[-i(u_{xx} + u_{yy}) - qi \ (|u|^2 + |v|^2)u] = 0, \quad (25)$$

$$v_t + p[-i(v_{xx} + v_{yy}) - qi\ (|u|^2 + |v|^2)\, v] = 0, \tag{26}$$

where $p \in [0,1]$ is an embedding parameter, we use it to expand the solutions in the following form:

$$u(x,y,t) = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \ldots \tag{27}$$

$$v(x,y,t) = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \ldots \tag{28}$$

The approximate solution can be obtained by setting $p = 1$ in eqs. (27) and (28):

$$u(x,y,t) = u_0 + u_1 + u_2 + u_3 + \ldots \text{ and } v(x,y,t) = v_0 + v_1 + v_2 + v_3 + \ldots \tag{29}$$

Now substituting from (27) and (28) into (25) and (26) respectively, and equating the terms with the identical powers of p, we can obtain series of linear equations as follows:

$$p^0: \ \dot{u}_0(x,y,t) = 0, \tag{30-i}$$

$$\dot{v}_0(x,y,t) = 0, \tag{30-ii}$$

$$p^1: \ \dot{u}_1(x,y,t) = i(u_{0xx} + u_{0yy}) + i\,u_0(|u_0|^2 + |v_0|^2), \tag{31-i}$$

$$\dot{v}_1(x,y,t) = i(v_{0xx} + v_{0yy}) + i\,v_0(|u_0|^2 + |v_0|^2), \tag{31-ii}$$

$$p^2: \dot{u}_2(x,y,t) = i(u_{1xx} + u_{1yy}) + i\ u_0(2|u_0||u_1| + 2|v_0||v_1|) + i\,u_1(|u_0|^2 + |v_0|^2), \tag{32-i}$$

$$\dot{v}_2(x,y,t) = i(v_{1xx} + v_{1yy}) + i\,v_0(2|u_0||u_1| + 2|v_0||v_1|) + i\,v_1(|u_0|^2 + |v_0|^2), \tag{32-ii}$$

The solution of equation (30) using the following initial conditions [22]:

$$u(x,y,0) = a_0(1 - \varepsilon\cos(\sqrt{2}\alpha\,(x+y))),$$

$$v(x,y,0) = b_0(1 - \varepsilon\cos(\sqrt{2}\alpha\,(x+y))).$$

Then, solutions of (31) are:

$$u_1(x,y,t) = \int_0^t [\,i(u_{0xx} + u_{0yy}) + i\,u_0(|u_0|^2 + |v_0|^2)]dt$$

$$= i\,a_0 t\,[4\alpha^2\varepsilon\cos(\sqrt{2}\alpha(x+y)) - q(a_0^2 + b_0^2)(-1 + \varepsilon\cos(\sqrt{2}\alpha(x+y))^3]\ ,$$

$$v_1(x,y,t) = \int_0^t [\ i\,(v_{0xx} + v_{0yy}) + i\,v_0(\ |u_0|^2 + |v_0|^2)]dt$$

$$= \ i\,b_0 t\,[4\alpha^2\varepsilon\ \cos(\sqrt{2}\alpha(x+y)) - q(a_0^2 + b_0^2)(-1 + \varepsilon\ \cos(\sqrt{2}\alpha(x+y))^3]\ .$$

By the same process we can find the solution of (32), and all other terms. To illustrate the advantages and the accuracy of the homotopy perturbation method for solving focusing Manakov system in two space variable, we have applied the method and using the first order perturbation, i.e. the approximate solutions are

$$u(x,y,t) \cong U(x,y,t) = u_0(x,y,t) + u_1(x,y,t), \tag{33}$$

$$v(x,y,t) \cong V(x,y,t) = v_0(x,y,t) + v_1(x,y,t). \tag{34}$$

We evaluate the numerical solutions using the first order perturbation from (29) with the constants, $a_0 = 0.08$, $b_0 = 0.1$, $\alpha = 0.05$, $\varepsilon = 0.05$, $q = 1.0$, $x_L = y_L = 0$, $x_R = y_R = 115$. The numerical results are represented in Figures (2,3) for the approximate solutions $u(x,y,t)$ and $v(x,y,t)$. The figures shows the behavior of the approximate solution at different time $t = 5, 10, 15, 20$ respectively , from the top to bottom and from the left to right in two figures. We achieved a very good approximation for the solution of the system. It is evident that the overall errors can be made smaller by adding new terms of the iteration formulas.

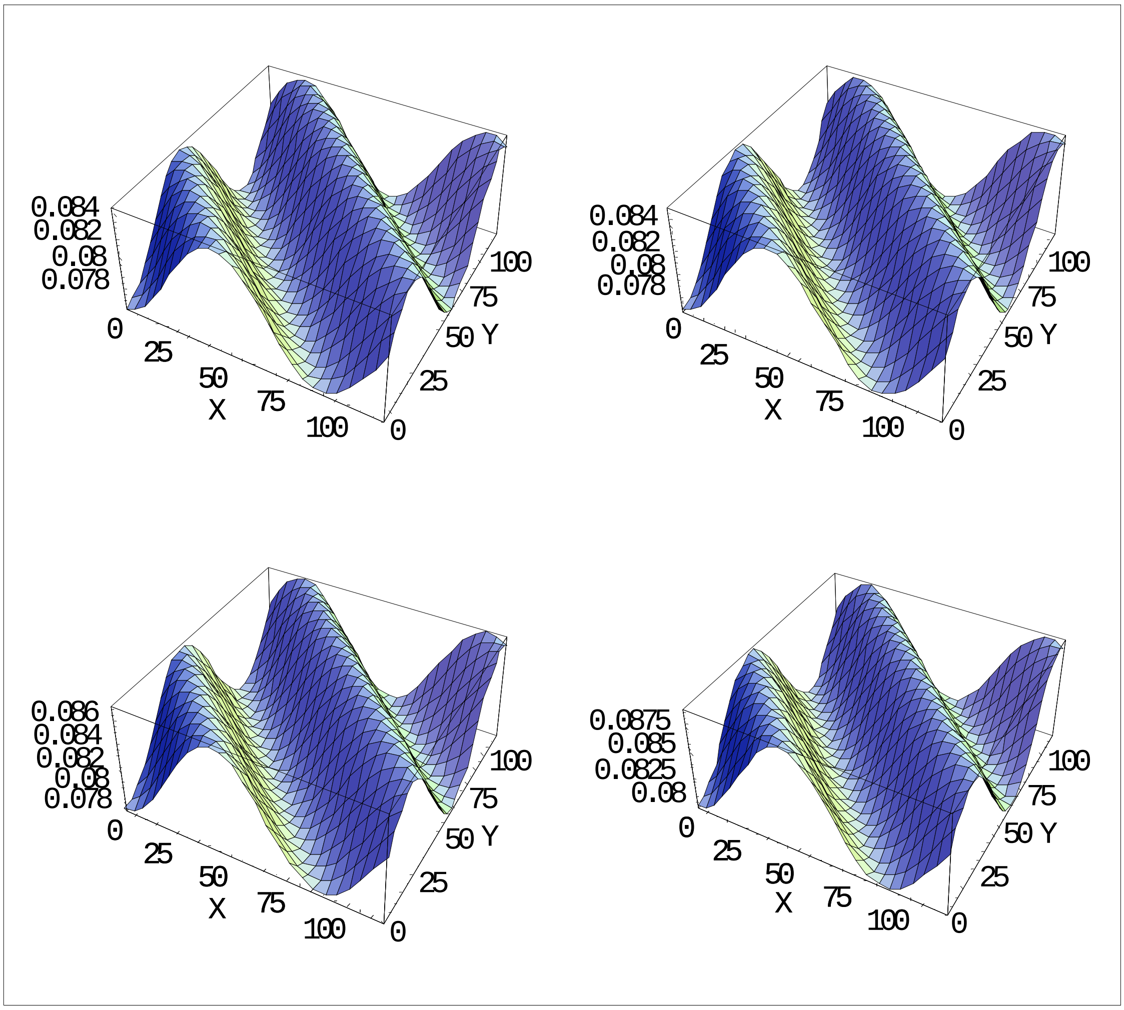

Figure 2: Long-time evolution of the wave solution |u(x, y, t)|

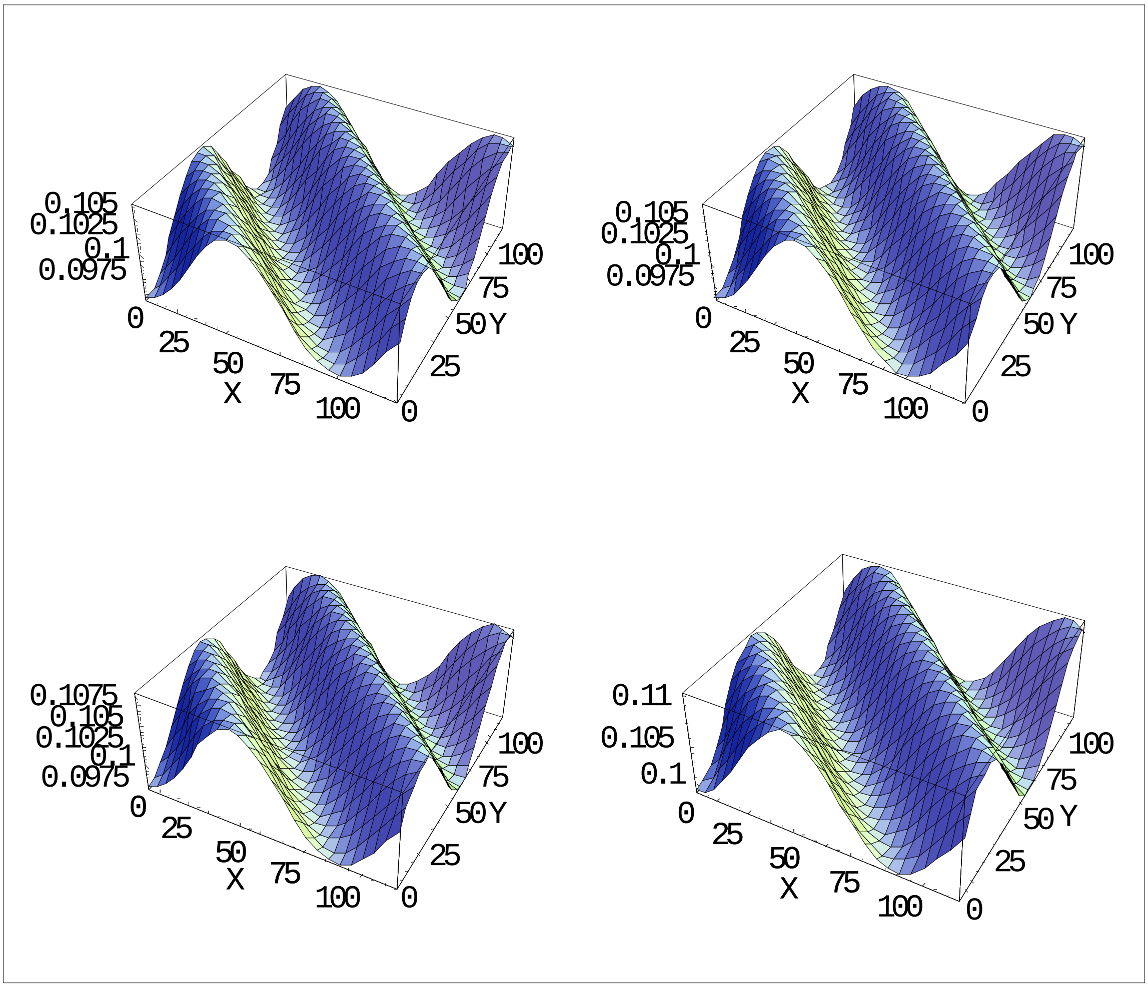

Figure 3: Long-time evolution of the wave solution |v(x, y, t)|

3.4. Conserved quantities in two space variable

In order to verify whether the proposed methodology lead to higher accuracy, we will use the same procedure as in section 3.2 we can emphasize that a good numerical scheme should have excellent long-time numerical behavior, as well as energy conservation property. To monitor the accuracy of the homotopy perturbation method, we consider the following two conserved quantities for the fixed value $y = y_0 = 115$,

$$E(u) = \int_{-s/2}^{S/2} |u(x, y_0, t)|^2 \, dx \quad \text{and} \quad E(v) = \int_{-s/2}^{S/2} |v(x, y_0, t)|^2 \, dx$$

Where $s = (2\pi / \alpha) = 40\pi$ (for $\alpha = 0.05$) is the spatial period of the solution [26]. Table 2 shows $E(u)$ and $E(v)$ for various times. The nearly constant values of both $E(u)$ and $E(v)$ show that the method is working well.

Time	E(u)	E(v)
3	0.802366	1.2537
6	0.808204	1.26282
9	0.817934	1.27802
12	0.831556	1.29931
15	0.849071	1.32667
18	0.870478	1.36012

Table 2

Following the stability analysis suggested by Tan and Boyd [23], the wave solution is linearly stable only if the perturbation wave number α is above the critical value $\alpha_c = \sqrt{2(a_0^2 + b_0^2)}$; otherwise the wave solution is unstable. For the our chose of the constants $a_0 = 0.08$, $b_0 = 0.1$, $\alpha = 0.05$, we find that α_c=0.181108 , therefore, the wave solution in this case is unstable. The amplitude of u and v undergoes oscillations between the near-uniform state and the one-hume state .

4. Conclusions

In this work, we proposed homotopy perturbation method for solving the focusing Manakov system of coupled nonlinear Schrödinger equations in one and two space variables. We achieved a very good approximation of the system by using first order of perturbation. A clear conclusion can be draw from the numerical results that the homotopy perturbation method provides highly accurate numerical solutions without spatial discretizations for nonlinear partial differential equations. The accuracy of the method is verified by ensuring that the conserved quantities remain almost constant. Finally, we point out that, the approximate solutions $u_i(\boldsymbol{x},t), v_i(\boldsymbol{x},t)$ are obtained according to the iterative equations by using Mathematica package (version 5).

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طريقة هوموتوبي المثارة لحل نظام مانكوف الغير خطى

في هذا البحث تم تطوير طريقة هوموتوبي المثارة لحل نظام مانكوف الغير خطى المكون من معادلات شرودنجر الغير خطية المرتبطة وذلك في كل من الفراغ الأحادي و الفراغ الثنائى. ومن مميزات هذه الطريقة أنها تتقارب بسرعة ولا تحتاج إلي حل نظام من المعادلات الجبرية الغير خطية وذلك لأنها لا تحتاج ألي تقسيم مناطق تعريف متغيرات المسألة. النتائج العدية أظهرت قوة وفعالية وسهولة الطريقة المعروضة لحل المعادلات الغير الخطية المرتبطة. ولقياس دقة الطريقة تم حساب الكميات المحافظة والتي وجدت انها دائما ثابتة.